Name: ________________
Student Number: ________________

Exam on WBPH030 “Solid State Physics”

Content: 5 pages (including this cover page)

Monday January 24 2022 (16:00-18:00)

Exam drafted by (name first examiner): Jianting Ye

Exam reviewed by (name second examiner): Graeme Blake

For administrative purposes; do NOT fill the table

	Maximum points	Points scored
Question 1	26	
Question 2	24	
Question 3	26	
Question 4	24	
Total	100	

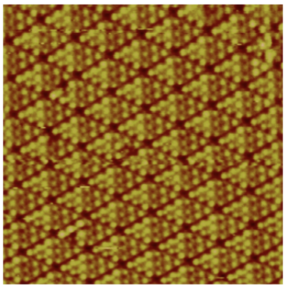
Final mark: _________

Question 1: Crystal structure and X-ray scattering

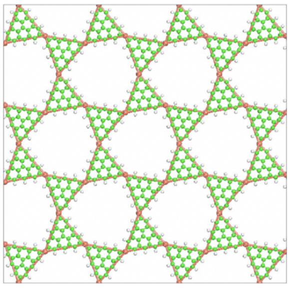
Note that a rough hand sketch is sufficient, don't spend your time making good-looking drawings!

(10p) Apply the idea of repeating environment to the following 2D patterns, identify the underlying Bravais lattice. Find the relationship between the translation vector **a**, **b**, and the between them (say |**a**| = |**b**|, and).

A

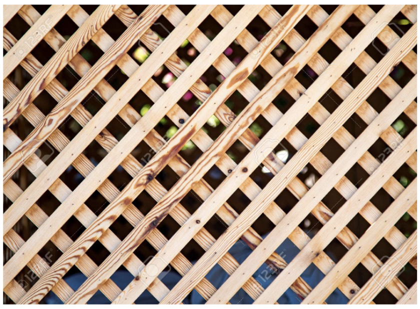


B

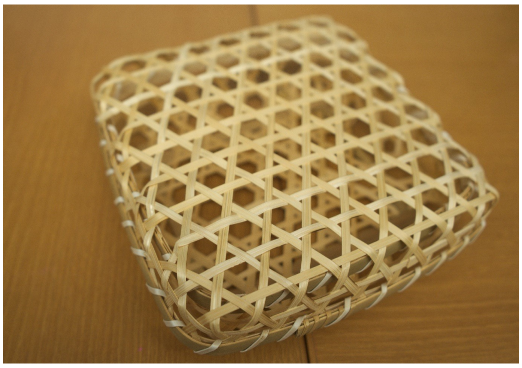


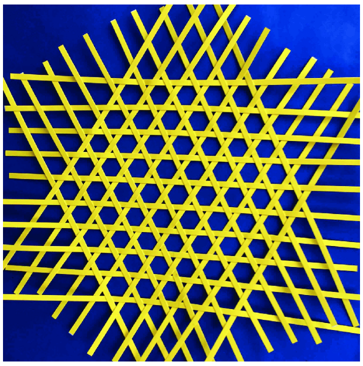
C

D



(3p) The so-called Kagome lattice shown below can be viewed as a 2D Bravais lattice. Identify the primitive cell of the Kagome lattice if each crossing point of two bamboo ribbons can be regarded as a lattice site. Count the number of lattice sites in a primitive cell.



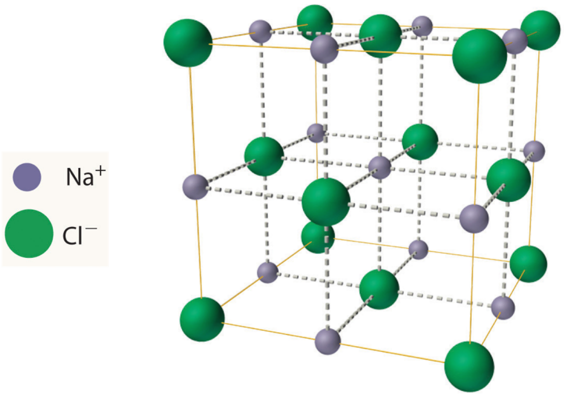


(5p)

a) Show that a reciprocal lattice vector $\boldsymbol{G} = h\boldsymbol{b}_1 + k\boldsymbol{b}_2 + l\boldsymbol{b}_3$ is orthogonal to the lattice plane (hkl).

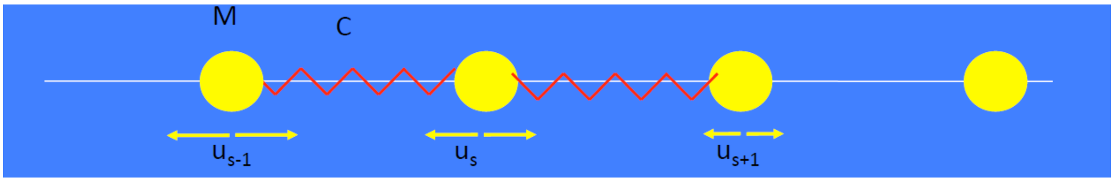
b) Show that the distance d_{hkl} of two lattice planes with Miller indices (hkl) is given by $d_{hkl} = 2\pi N / |h\boldsymbol{b}_1 + k\boldsymbol{b}_2 + l\boldsymbol{b}_3|$.

(8p) By analyzing the structure factor of NaCl, show the lattice planes *h,k,l,* from which diffraction peaks are expected. (*hint*: NaCl has two kinds of atoms in a cubic system, which can be considered as 4 Na at 000 FCC structure + 4 Cl at ½ ½ ½ FCC structure).



Question 2: Phonons and thermal properties.

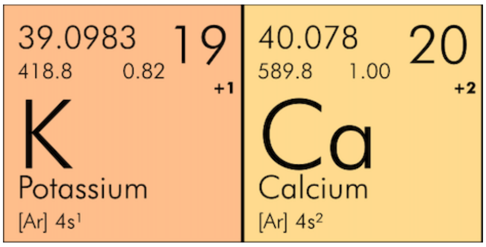
As shown below, consider a linear chain of N atoms all with mass M, and the force constant is C.



1) (8p) Calculate and sketch the dispersion relation in the first Brillouin zone for the chain above.
2) (4p) Sketch the dispersion relationship in the first Brillouin zone if either the force constant or the mass of atoms changes. The mono-atomic chain becomes a di-atomic chain. How will the $\omega(k)$ relationship evolve?
3) (6p) Suppose a green laser light (532 nm) illuminate the chain. Based on the sketch in the 2), estimate which vibrational mode(s) can be excited?
4) (6p) Suppose the optical branch has the form of near $K = 0$, where is a constant. In three-dimension case, show that for , and , for all .

Question 3: Free electrons in potassium and calcium.

Consider two crystals: K (1 valence *e*) and Ca (2 valence *e*). Both are 3D simple cubic. Suppose we can slice the single crystals and isolate atomic planes, both K and Ca can form 2D square lattices of constant *a* and *b*, respectively



1) (6p) Calculate the 3D density of state ($D_{3D}(E)$) and 2D density of states ($D_{2D}(E)$), We assume the size of the specimen is L^3 and electron mass is m_e. ($L = N_x a$ or $N_x b$ for K and Ca, respectively).

2) (12p)
   a) Argue why both of them are metals (use the concept of Brillouin zone and Fermi surface).
   b) Calculate and sketch the size and shape of the Fermi surface for the 2D K and Ca (assume no modification due to the periodic potential of K^+ and Ca^+).

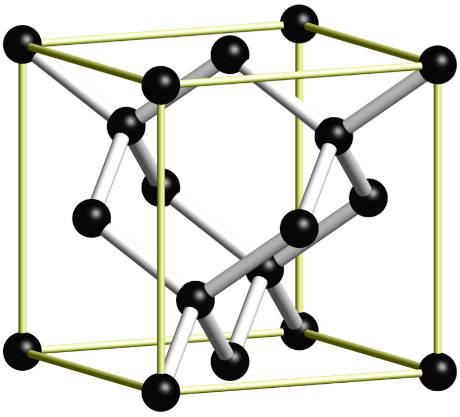
3) (8p) Suppose the scattering time τ is identical for K and Ca. From Drude's model briefly compare the difference between 2D K and Ca in electrical conductivity and thermal conductivity $K = ⅓\, c_{el}\, v^2\tau$, and their ratio $K/\sigma T$ (which is called the Wiedemann-Franz law).

$$\underline{C_{el} \quad \frac{{}^2 n k_B^2}{2E_F} T \quad \mathrm{JK^{-1}m^{-3}}}, \underline{\vec{j} \quad \frac{ne^2}{m_e} \vec{E}}$$

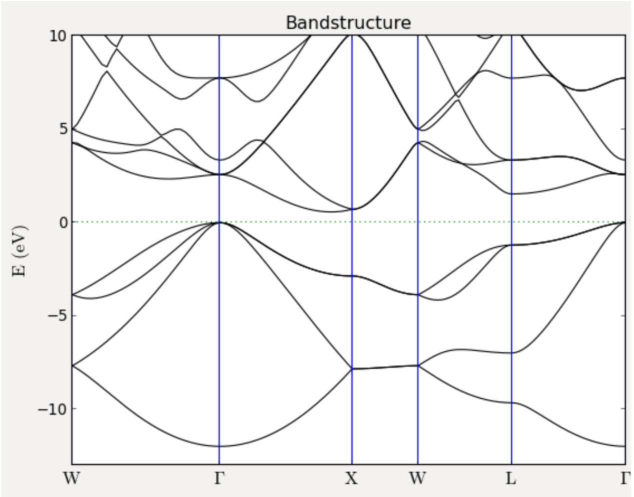
v is the velocity of electron, $n = N/V$, $mv_F^2 \approx 2E_F$, where v_F is the Fermi velocity.

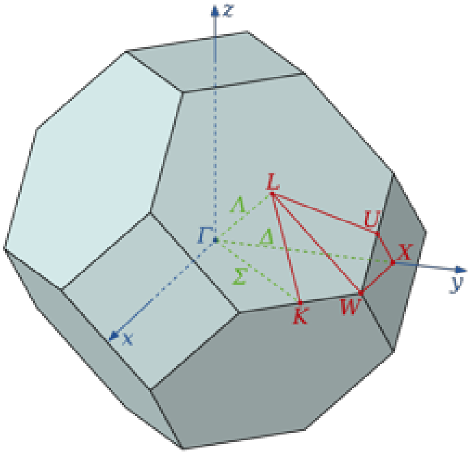
Question 4: Semiconductor and superconductivity

Silicon (Si) is arguably the most important material of the last century, which is frequently referred as the Silicon age.



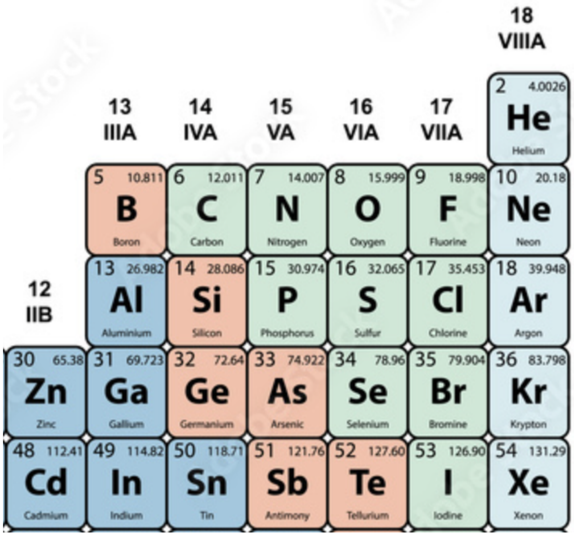
Crystal Structure





Wigner-Seitz cell in the K space

E (*k*) relationship of Silicon



12 IIB	13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA	18 VIIIA
						2 4.0026 He Helium
	5 10.811 B Boron	6 12.011 C Carbon	7 14.007 N Nitrogen	8 15.999 O Oxygen	9 18.998 F Fluorine	10 20.18 Ne Neon
	13 26.982 Al Aluminium	14 28.086 Si Silicon	15 30.974 P Phosphorus	16 32.065 S Sulfur	17 35.453 Cl Chlorine	18 39.948 Ar Argon
30 65.38 Zn Zinc	31 69.723 Ga Gallium	32 72.64 Ge Germanium	33 74.922 As Arsenic	34 78.96 Se Selenium	35 79.904 Br Bromine	36 83.798 Kr Krypton
48 112.41 Cd Cadmium	49 114.82 In Indium	50 118.71 Sn Tin	51 121.76 Sb Antimony	52 127.60 Te Tellurium	53 126.90 I Iodine	54 131.29 Xe Xenon

1) (4p) Silicon has fully-filled valence bands and a band gap > 1eV. Briefly explain how can the intrinsic carriers, electron *n* and hole *p*, be created in Si at a finite temperature.

2) (4p) From the periodic table shown above, choose the elements for *p*- and *n*-type doping in Si, draw the schematic diagrams at the band edge (for *p*- and *n*-type). And identify the locations of the impurity bands in the Wigner-Seitz cell for *p*- and *n*-type dopings.

3) (3p) What will happen on the interface if you now bring *p*- and *n*-type Si in contact?

4) (4p) Sketch and explain the rectification behavior of a *pn* junction.

5) (5p) Draw the energy levels at the interface before and after diffusive equilibrium is established. Based on the energy diagram explain how a solar cell works.

6) (4p) With very strong p doping, when dopant replaced 9% of Si, superconductivity was discovered at 0.35 K. What are the two hallmark physical properties to be expected for T < 0.35 K?

--------------------------------- End of Question ---------------------------------